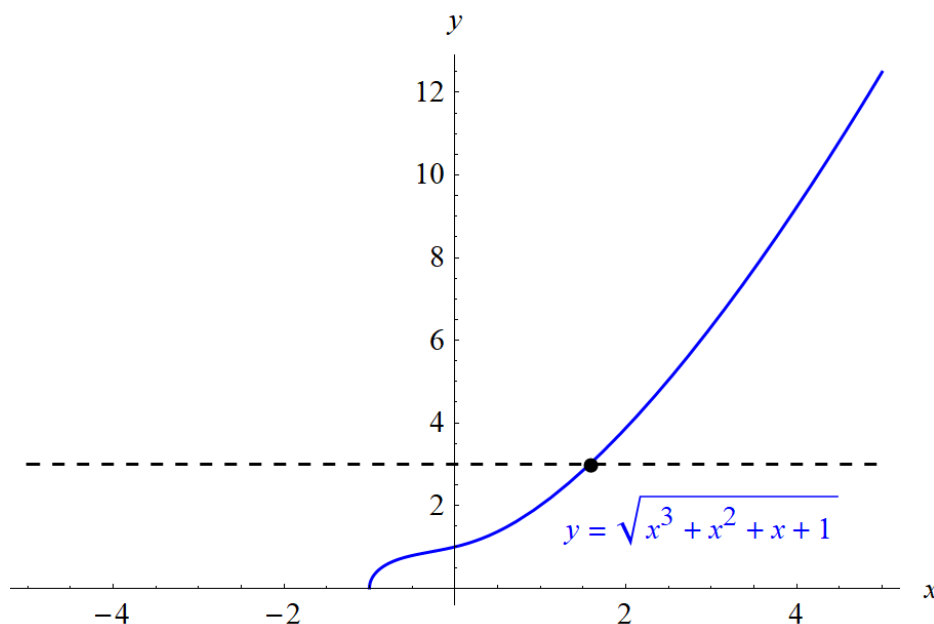


Exercise 59

Graph the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$ and explain why it is one-to-one. Then use a computer algebra system to find an explicit expression for $f^{-1}(x)$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

Solution



Because every value of x on the curve corresponds with exactly one value of y , the function is one-to-one; that is, the function passes the horizontal line test. Using the CAS yields

$$f^{-1} = \frac{1}{6} \left[-2 - 4 \left(\frac{2}{-20 + 27x^2 + 3\sqrt{48 - 120x^2 + 81x^4}} \right)^{1/3} + 2^{2/3} \left(-20 + 27x^2 + 3\sqrt{48 - 120x^2 + 81x^4} \right)^{1/3} \right]$$

for one of the three inverse functions. The other two involve complex numbers, and we don't care about those.